QTris: The Game of Quantum Mechanics

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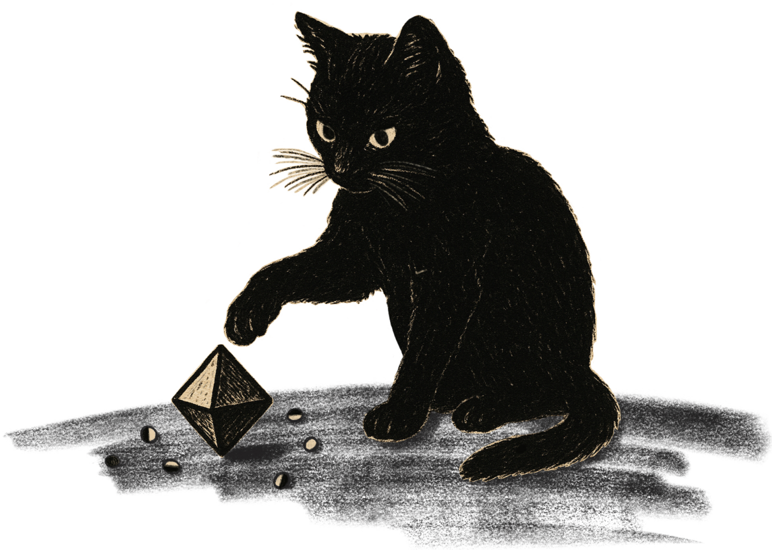

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Preface to the online edition

For a long time, I wanted to make a game that (i) used the rules of quantum mechanics, and (ii) was a good game to teach them. For a long time, I also refrained from trying because how could I do anything that in some way was not already done by J. Eisert [EWL99] or W. Dür [LD19]? Then, somehow I felt that there was still room for something that would be useful for teaching quantum mechanics at high school level or to the general public that would start from the conceptual building blocks of quantum mechanics. So I made QTris and started using it to explain quantum mechanics to a lay audience. I thought it was useful because it refrained from useless metaphors: quantum mechanics had to be understood through the understanding of its rules and of its consequences, and because the consequences are those that would give meaning to the most counter-intuitive parts of the theory, like coherent superpositions or entanglement.

This book contains all the game rules. They are interspersed with *leaflets of quantum mechanics*. They are simple explanations that connect the game rules to quantum mechanics at the level of a popular science book. The text also often makes reference to other materials that my group and I use to teach in schools. Those parts are still in a testing phase, and are not included here. However, I decided not to remove those references in order to show the scope of the full project.

The online downloadable version of the QTris game is a way to share this set of rules with the community of physicists interested in quantum mechanics research, its teaching, and its playing. This material has been copyrighted but I am happy if people use it. I am also very grateful if anyone would contact me to warn me about mistakes, or simply wants to discuss the game mechanics and possible improvements.

If you decide to use this game for fun or didactical purposes, please credit the authors and point to the webpage where you can find its up-to-date version and more materials at `www.quantumphysics.fun/qtris`.

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Chapter 1

Introduction

Welcome to QTris, a board game aimed at learning the fundamental concepts of quantum mechanics (QM) in a rigorous way!

1.1 How to use this book

This book constitutes the first part of a larger and more ambitious project aimed at teaching quantum mechanics through gaming. The first part of the project, that is, the material you are reading now, contains the game rules of QTris. The second part, which will be distributed in other ways, contains a rigorous explanation of the formal structure of quantum mechanics that is aimed at high-school level. Some of the text in this first part will sometimes make reference to the second part, but those references are not necessary to learn how to play the game and get a first grasp of quantum mechanics.

1.2 Game philosophy

Why a board game to learn quantum mechanics? The simple answer could be that by gaming one can entice young students to a difficult discipline like quantum mechanics and make it entertaining. Now, we are not sure we subscribe to this pedagogical view. First, because there is something dishonest in the travesty of something complicated as something simple. Second, for like every child knows, games[1] are a very serious and difficult thing. As such, a game is the right tool to learn something serious and difficult like quantum mechanics. And thirdly, because the goal of studying should not be to learn more things, but to learn how to give full attention. Diversions are a way to, indeed, divert it.

One of the difficulties in teaching or learning quantum mechanics lies in the fact that its rules are clear as long as they are abstract. When one then

[1] that others call Mathematics.

tries to connect the abstract theory to something *concrete* or to common sense, things get murky. The request of concreteness should not be dismissed. On the one hand, this is the request of understanding how the abstract rules work and connecting them to the facts of experience. In trying to understand Nature, one should hold this request in the highest regard. On the other hand, the request for concreteness is more often tainted with a sentiment that is in fact detrimental in our relationship with the understanding of Nature, that is, pure hostility towards abstraction — according to the spirit of the times. We believe that the hostility towards abstraction is part of the more general hostility towards attention. Moreover, since abstraction is difficult, it will always result in poorer evaluations in a school context. Apparently, the goal of modern education is to find a way to show in a table or a graph an improvement in some kind of scores. For instance, one can show that children understand and can use the notion of *sticks* better than that of *segments*, so better do away with segments and teach them un-math instead of mathematics.

Of course, our point of view is that one of the aims of Education is indeed exactly that of teaching how to find a structure in the facts of the world, in other words, to teach how to abstract.

The reason why the rules of QM become ambiguous when one tries to make them concrete — or, even worse, to visualize them — is that QM is telling us that Nature behaves fundamentally in a way that is quite far from our intuitions and our philosophical prejudices. To be sure, such intuition and such prejudices are there for a reason, the reason being that in most of our interactions with the external world, they seem to give a good account of it: ostensibly there are objects, ostensibly they have an effect on each other, ostensibly they are there regardless of our sense-perception of them. However, as our technical capabilities allowed us to investigate microscopical phenomena, many of the notions we were used to just started to crumble. Trying to understand QM in terms of our usual mental categories gets then tied up in interpretational questions in which many generations of physicists and philosophers of QM got lost. These questions are so complicated that some believe they are intrinsically unsolvable. The vast majority of quantum physicists, though, got to terms with just using the abstract rules, and, for the most part, understanding how they connect to the experiments they can carry on. Thanks to the correct application of the abstract rules to the concrete experimental situations they are able to attain an incredible explicative and predictive power from QM: it has been said many times that there is no theory of Physics that is more successful than QM.

Of course, it takes a lot of experience and practice to be able to use quantum mechanics in a correct way. Even seasoned practitioners and researchers may experience a hard time. At some point, with experience, scientists gain some level of confidence about how to inquire what quantum mechanics is going to tell them about some given physical situation. As a matter of fact, at that point they tend to resort to the abstract rules. They know they work, they use them in an abstract way, and perhaps only at the very end of their chain of calculations they will try to interpret their results in some physical and perhaps intuitive way.

In QM, these problems seem so hard that there is no wonder that this subject is not really taught at high school if not at a very superficial level. The student is supposed to learn the atomic structure or the whole story about quantization of energy or the Schrödinger equation. These are all things that students cannot really grasp because they involve higher mathematical tools like partial differential equations, usage of metaphors like “probability waves”, together with a very large amount of physics that is thereby involved. This approach does not lead very far. It does teach some verbal habits like being able to answer some questions like «What is the atom structure like?» by saying «Electrons possess discrete values for the energy» without really understanding how and why. All this knowledge does not touch any of the essential aspects of QM. It is some sort of zoology about the behavior of elementary particles.

To this critique, the typical answer of the community of physics education is that young students need to be *exposed* to certain ideas, and that only later on they will be able to understand things at a deeper level during their university studies.

It may as well be that these people are right. That there is not a way to make the essential aspects of QM accessible at high-school level. Were this be the case, this little tentative will fail in its most ambitious goal. And yet, something tells us that it is exactly the young students that can get the greatest payoff from an abstract exposition. The reason is that the young love to play games, and gaming, in its very nature, is abstract. The rules of a game, say, chess, or any other board game, are abstract. The players know very well that the concrete interpretation of the tiles and the pieces on the game board is immaterial, that what really counts is how the pieces move, and their mutual relationship on the game board given by the rules. All that counts is how to devise a strategy that, within those rules, leads to winning. Whether the piece resembling a unicorn is really a unicorn is not important at all. The unicorn-shape is a make-believe, a myth, that players know very well should never be taken into account in their strategy. If someone asked, typically an adult, a question regarding the unicorn and whether a unicorn should not rather do something else on the board, this one would be told that they have not understood the game. Kids, on the other hand, who take play seriously, know the function of the pieces very well. Also some adults know that, when they get some relief from the immediate material needs of their life. The human being is a *homo ludens* and play is an essential condition for the formation of culture.

In making contact with experience, QTris and the formal rules of QM show that at the very least these give a correct description of the behavior of certain instruments when they interact with a given class of physical systems, typically microscopic. This does not mean that we are proposing an instrumentalist point of view. All we want to say is that — although things might be much more complicated than we know — experience revealed to us a collection of facts and that these facts can be organized very well within a the set of the abstract rules of QM.

1.3 How to use this material

The material of this book can be used in a multitude of ways. It is fundamentally divided into two parts. In the first part, that is, the material presented here, you learn the game. In the second part, which is not included here, you learn some mathematical tools and the formal structure of quantum mechanics rigorously. By the end of the second part, you will have understood that playing QTris is effectively performing calculations in quantum mechanics, and that doing calculations in quantum mechanics corresponds to playing a certain type of game in QTris. In its entirety, this material can cover an entire course of quantum mechanics at high school or pre-university level, which can cover up to fifty hours of lessons, including gameplay and exercises. Part of these hours consists of teaching mathematical concepts that are already part of the curriculum of — at a minimum — the scientific high school. The material is thus organized by parts and number of teaching hours required to administer them.

1. Part I: The rules of the game (this book)
   - Basic rules of the game to be able to play (Chapter 2): $\leq$ 2 hours
   - Basic quantum mechanics within the game (Chapter 2): 3 hours
   - Advanced gameplay (Chapter 3): 2 hours
   - More advanced notions of quantum mechanics (Chapter 3): 3 hours

   In particular, the concepts of quantum mechanics that appear in the text are presented in the following yellow sheets.

   > QM
   >
   > **Leaflets of quantum mechanics.** The yellow leaflets with the symbol "QM" like this one explain how some game component or rule should be understood in QM. It is not necessary to read them to learn how to play.

2. Part II: Quantum mechanics (not included here)
   - Matrices: 4 hours
   - Probability: 4 hours
   - Complex numbers: 6 hours
   - Dirac notation: 6 hours
   - History of quantum mechanics: 2 hours
   - Postulates: 4 hours
   - Pure and mixed states: 2 hours
   - Operations: 3 hours
   - Entanglement: 3 hours
   - Quantum evolutions: 3 hours
   - Experiments and algorithms: 4 hours

1.3.1 Learning objectives

The learning objectives of this material are two. First, by learning how to play the game, one learns the fundamental formal structure of quantum mechanics, the notions of state, operations and measurement, as well as quantum mechanics as a probabilistic theory, the difference between quantum states and classical probability distributions, entanglement and its differences with respect to classical correlations. The reaching of such objectives allows a comprehension of the fundamental concepts of quantum mechanics, which is as free of ambiguities and metaphysical prejudices as possible.

By utilizing the second part of the book (not included here), we aim at teaching in a deeper way the formal apparatus of quantum mechanics. This will give the student the ability to perform calculations, predictions and solve problems.

The mathematics utilized is at high-school level. Sections or exercises marked by * are more advanced and can be skipped without compromising the understanding of later material.

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Mathematical examples. In the second part of this book (not included here), we will develop the formal structure of quantum mechanics. This is expressed in terms of mathematical tools such as matrices, complex numbers, and probabilities. Many mathematical examples are inserted into boxes like this one.

Exercise.

This kind of boxes denotes exercises that the reader is invited to solve.

1.4 Game structure

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The structure of this game is made of three phases that reproduce the structure of quantum mechanics: *preparation*, in which the game board is set together with the tiles and cards. The second phase, called *operations*, in which the players play cards to change the disposition of the tiles on the board, and finally the phase of resolution, in which points are assigned: this phase will always be called in QTris as the *measurement* phase.

The objective of the game is the same as tic-tac-toe: to line up three identical symbols within a grid of nine squares. QTris, however, has several differences from the usual tic-tac-toe. The game begins by preparing the grid with one tile per square. At the end, in the *measurement* phase, after rolling a die each tile will have a certain probability of becoming the tile that assigns points to one or another player. In the game phase called *operations*, players can play cards to modify the tiles on the board in order to improve their chances of winning when assigning points. Surprisingly, the game could end with more tic-tac-toes simultaneously on the grid!

The differences, however, do not stop here. By reading the following pages, you will learn all the rules of the game, and, together with these, the peculiarities of quantum mechanics. Indeed, we can declare without fear of contradiction that there is no fundamental difference between QTris and quantum mechanics. Everything that is possible in quantum mechanics will also be possible in QTris.

1.5 The structure of quantum mechanics

As we shall repeat several times, the structure of QTris is the same as of quantum mechanics. It consists of three phases. The first phase is where the game board is prepared. This consists of applying a certain protocol to determine what tile to place on each square. The second phase is that of *operations*, in which players play cards that change the tiles on the squares. The third phase is that of *measurement*, in which the possible outcomes on every square is random, but with very precise probabilities given by the rules of QTris. Players perform operations in order to obtain more favorable probabilities for their score.

QM

The structure of a quantum experiment. In quantum mechanics, every experiment consists of three phases: *preparation, operations,* and *measurement*. In the preparation phase, it is determined what the initial state of the system is. This happens through a series of physical operations, for example, by passing silver atoms at a certain temperature through a collimation apparatus which sends them into a narrow beam. The second phase (operations) consists in the temporal evolution of the system because of the interactions in the system or operations that the experimenter or the environment perform on the system. Finally, the phase of measurement reveals in the lab the effect of an experiment. In this phase, a measuring apparatus is in contact with the system and following this interaction it records a result of the measurement, see Fig.1.1. For example, if we perform an experiment with photons fired at a screen with two slits, the preparation consists of the way the photons are shot, the operations consist in the interaction of the photons with the screen and the slits, and the measurement consists in seeing in which point the photons that went through the slits hit the revealing plate.

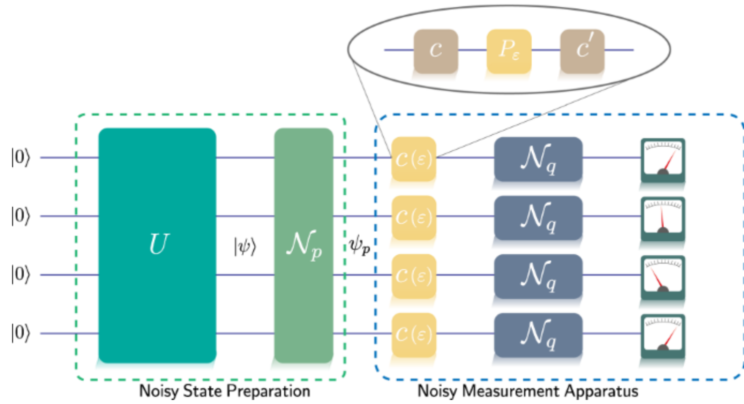

Figure 1.1: **The structure of a quantum experiment.** The first layer with all the symbols $|0\rangle$ prepares the initial state of the system in all the qubits being 0. The second box containing U and $\mathcal{N}_p$ shows the phase of operations where the state is manipulated, and the last box shows the detail of the measurement phase. From [OLHL22].

QM	**Probabilities in quantum mechanics.** In a quantum experiment, which of the possible outcomes does indeed occur is random. The probabilities of such events, however, can be calculated *exactly*. The whole point of the theory of QM is to give a tool to compute the probabilities of getting certain results in certain experiments. An experimenter who has access to a sufficient number of operations between preparation and measurement may change these probabilities in the desired way, and this is at the basis of applications like quantum technologies and quantum computing.

Chapter 2

The Game

2.1 Number of players

The game is suitable for $2-9$ players aged 12 and up. In the basic version, the two players will be referred to as Alice and Bob. Alice and Bob can also be played by a team that decides the operations together. In advanced versions, a third player is called Eve. Alice and Bob can cooperate against Eve or everyone can play against everyone. In the case of three teams (Alice, Bob, Eve) of three people each, the number of 9 players is reached.

2.2 Game components

The QTris game is made up of the following components:

- The game grid, divided into nine squares.
- Circular tiles: white ○, black ●, and black-and-white ◐, ◑.
- Yellow triangular tiles △, ▽, △, ▽.
- Quadrangular tiles H, U
- Two ten-sided dice (d10): one for tens and one for units.
- One deck of cards, corresponding to quantum operations and denoted by the symbols

$$I, X, Y, Z, H, U, C_X$$

2.2.1 Game grid

The game board, shown in Fig. 2.1, is a board made up of nine numbered squares. On each square you can place a game tile. The cards are played on

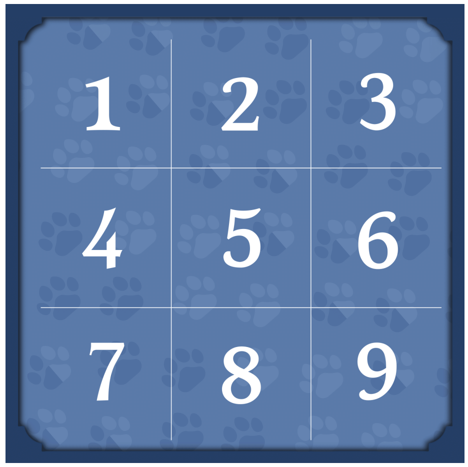

Figure 2.1: **Game grid.** Every square represents a quantum system called *qubit*.

the squares and transform the game tiles into other game tiles, according to the rules described in Sec. 2.3.

QM

Squares as quantum systems. As a first explanation of how quantum mechanics is represented in QTris, let us talk about the game board. The board represents a composite quantum system! In fact, every square represents one quantum system, called *qubit*, and thus the entire board represents a quantum system composed of nine subsystems, or nine single *qubits*. Any quantum system can be described by a system made of a suitable number of qubits.

In QTris, the core game mechanics operates primarily at the level of one or two qubits. This means that the available actions can be applied either to a single square or to a pair of squares. As we know, however, the game board has nine qubits.

2.2.2 Game tiles

The game consists of tiles in three distinct shapes: circular, triangular, and quadrangular. Each of them represents a *quantum state*, on one or more qubits.

QM

Tiles as quantum states. In quantum mechanics, every physical system can be described by a construct called *state*. The state of the system allows to predict the probabilities for the outcomes of every possible experiment on that physical system. You got it right: *probabilities*. Nature only gives out probabilities for what may happen. However, such probabilities are not arbitrary, but follow very precise physical laws. These are the laws of quantum mechanics. In QTris, every configuration of tiles on the game board represents the state of a quantum system of nine qubits. A tile on a single square of the grid is a state on one qubit.

Circular tiles

The simplest tiles in the game of QTris are the circular ones. Circular tiles come in three variants: white ○, black ●, and black-and-white ◐, ◑. Every time one places a single tile in a square, the corresponding state is called *pure*.

QM

Pure states. In quantum mechanics, every physical system is initially prepared in a certain state following an experimental protocol. If this protocol is both perfectly determined and executed, then the physical system is effectively prepared in a well-determined state said *pure state* (aka *kitten*). These are states that contain the maximum possible information about the system. In QTris, a circular tile on a square corresponds to a pure state of a single qubit.

So, in QTris, whenever you place a single circular tile on a square, you are choosing a pure quantum state for that qubit. You are saying: «This is what I know about this tiny piece of the system», and this knowledge is complete.

Circular tiles in QTris possess two fundamental properties: *color* and *orientation*. States ○, ● have a well-defined color, white or black, while black-and-white states ◐, ◑ have a well-defined orientation: they are oriented to the left or to the right, depending on the position of the black and white regions.

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The orientation of circular tiles in QTris is very important, but not for all of them. The solid tiles ○, ● can be rotated as one whishes, and they look exactly the same. For them, orientation is unimportant; a mathematician would say that they enjoy a circular symmetry. On the other hand, black-and-white tiles can be oriented with the black part on the left or right ◐, ◑ and represent different states (and physical properties). It is therefore very important to think about how tiles are oriented, to understand in the correct way how the operation cards can be played or the effect of a measurement on such states.

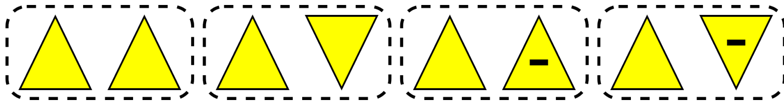

Figure 2.2: **Entangled states.** Each entangled state is a state of two squares. A triangular tile is placed on both squares of an entangled state. The triangular tiles of an entangled state must be of the same color.

QM

Schrödinger's kittens. The black and white tiles ◐, ◑ represent what are also referred to as *Schrödinger's kittens*. These cats do not have a well-determined color until one observes it. One can say that the color is that property that is summoned by the observation of color. However, once the color is observed — say, black — then it is determined as black and every further observation will show us that it is indeed black.[a] One can say that the ◐, ◑ kittens have a probability $1/2$ of being observed as white or black, while the ○, ● tiles have a probability 1 of being white or black, respectively. However, these kittens are not *just* probabilities: there is something more to it which is very important and that we will understand as we proceed with the game. If at the end of this rulebook and after having played a few games one will have understood Schrödinger's kittens, then one can say to have actually understood quite a bit about quantum mechanics!

[a]In the original example, Schrödinger talked about the property of being alive/dead instead of the color being white/black, which was a bit more dramatic.

Triangular tiles: entangled quantum states

In the game, it is possible to correlate some circular tiles with some other circular tiles by applying on them the C_X card, whose action is further clarified in Sec. 2.3.2. Triangular tiles are used to represent the *entangled* tiles, and the possible combinations of triangular tiles are shown in Fig. 2.2. The squares of an entangled state do not necessarily have to be adjacent: one can apply a card that creates entanglement on two distant squares of the game board. Entangled tiles are deeply connected, so much so that if you check one, you will instantly learn something about the other. When the triangles are oriented in the same verse, the two tiles are bound to give the same result upon measurement (correlation), while when they have opposite orientations, they are bound to yield opposite measurement outcomes (anti-correlation). Some of the triangles also feature a minus symbol ($-$), which represents the so-called *phase* of the entangled state. The concepts of correlation and phase are clarified in more detail in in Sec. 2.3.2, Sec. 2.3.3, and in Sec. 3.3, after introducing the action of the C_X operation card, which is responsible for creating entanglement, and exploring how entangled states behave under measurement.

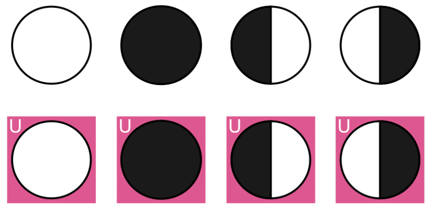

Figure 2.3: **Single square tiles available in QTris.** Each single-square tile corresponds to a game tile in circular shape. A circular tile can be decorated with a pink square U tile by playing the respective operation card. The way in which such states can be created within the game is described in the advanced rulebook, in Sec. 3.2

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Quadrangular tiles: more states in quantum mechanics

It is possible to decorate circular tiles with a quadrangular tile underneath, after using the respective U card. By "decorate", we refer to placing the quadrangular tile under the circular tile present on the square where we use the respective card, as shown in Fig. 2.3. Details on the effect of the operation cards are presented in Sec. 2.3.2. Decorated one-qubit tiles are part of the advanced version of the game and are illustrated in Sec. 3.2. Triangular tiles can also be decorated with quadrangular tiles , . The complete list of all possible representations of entangled states present in the advanced version of QTris is shown in Fig. 3.4 and Fig. 3.5.

2.2.3 Operation cards

The game includes a deck of fifty-two (52) operation cards. Each operation card represents an operation that can be performed on a state within the game. Essentially, the operation cards allow the player to act on a state to modify it. Thus, through the operation cards, the player can alter the arrangement of tiles on the game grid.

QM

Quantum operations. In quantum mechanics, the states of a system may change with time. This change is called evolution and it is called *unitary* or *non-unitary* depending on the laws that describe it. An evolution is enforced by a *quantum operation*; thus, a quantum operation has the effect of making the state evolve. In QTris, all of the operation cards describe the laws of the unitary operations on a quantum system. Unitary evolutions are reversible: they preserve the total probability and describe isolated quantum systems evolving without external interference. Non-unitary evolutions, on the other hand, arise when the system interacts with an environment or undergoes a measurement.

The operation cards available to players in the basic version of QTris are:

$$I, X, Y, Z, H, C_X$$

2.3 Game rules

In the following subsections we describe in detail the three phases of the game.

2.3.1 Game preparation

The preparation phase begins with randomly arranging the tiles on the game board, so that each space is occupied by a single tile.

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During preparation, only valid states within the game can be placed on the game board. Therefore, it is not possible to place only one of the two tiles that make up any entangled state.

In the base version, the tiles to choose from and place initially on the game board are ○, ●, ◐, ◑. We then proceed to build the deck of operation cards. In this rulebook, four possible preparations of the deck of operation cards are provided, which correspond to four different levels of difficulty of the game. The deck builds are:

Card	Base	Advanced
I	5	5
X	10	10
Y	5	5
Z	10	10
H	12	12
C_X	10	10
U	0	9

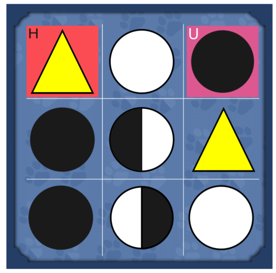


Figure 2.4: **Game preparation.** Example of a game board configured with both entangled states and decorated tiles , .

The deck of operation cards is shuffled and four (4) cards are dealt to each player. The players must now decide on their turns. To do so, both players must roll a ten-sided die (d10). The player who rolls the highest value on the die will play first, and will choose between the white tile symbol or the black tile symbol as their game symbol. The goal of the game is to make one (or more) tic-tac-toe (or *qtris*) using the chosen tile.

> **!** The only valid *qtris* are those made up of undecorated, single-colored tiles. For example, in the following grid there are *three* qtris for the black.
>
> $$\begin{pmatrix} \bullet & \circ & \circ \\ \bullet & \bullet & \circ \\ \bullet & \bullet & \bullet \end{pmatrix}$$

The players, now with four (4) operation cards in their hand, may decide to discard any number of operation cards from their hand and draw the same number from the top of the operation card deck. This game mechanic is called *mulligan*, and it is useful for ensuring that at the beginning of the game you have the operation cards in your hand that are appropriate for the game strategy you want to adopt.

In Fig. 2.4, a game setup involving both basic and advanced tiles is represented. On the game board, we can observe: a white tile ○ on squares nn. $2, 9$; a black tile decorated with a pink U tile on square n. 3; a black-and-white tile oriented to the right ◑ on square n. 8; a black tile ● on squares nn. $4, 7$; a black-and-white tile oriented to the left ◐ on square n. 5. Squares n. 1 and n. 6 are occupied by entangled tiles. The triangular tile on square n. 1 is decorated with a red H tile .

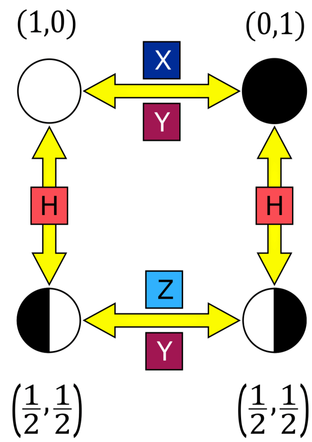


Figure 2.5: **Square map of single-square operations** X, Y, Z, H **and of measurement probabilities.** The numbers in the parentheses (x, y) next to the symbols ○, ●, ◐, ◑ represent the probabilities of obtaining ○ and ● respectively, after performing a measurement, as explained in Sec. 2.3.3.

2.3.2 Operations phase

The operations phase starts immediately after the completion of the preparation phase, and it is organized in game turns. It consists of playing the operation cards, which allow the players to change the state of the system.

The operations phase consists of ten (10) game turns, five (5) for each player. In each game turn, the player who is playing must:

1. Pick two (2) operation cards from the operation card deck.
2. Play two (2) operation cards from their hand.

The operations phase ends when both players have played five (5) turns, one each starting with the player who begins the game.

Let us now illustrate the effects of all the cards listed above.

Single-square operation cards

- The I card has no effect and is used to pass the turn.
- The operation cards X, Y, Z, H act on a single square, and the rules that determine their action are graphically represented in the square map of Fig. 2.5. In particular, the X card acts as a *change color*, the Z card as a *change orientation* and the Y card both as a *change color* and as a *change orientation*, depending on the state on which we apply it. The H card acts as a *change between color and orientation*.

Exercise 2.3.1: Card play mechanics.

Show how to transform the game board prepared as:

$$\begin{pmatrix} ◐ & ○ & ○ \\ ● & ● & ○ \\ ● & ◑ & ● \end{pmatrix} \text{ into: } \begin{pmatrix} ● & ○ & ○ \\ ● & ● & ○ \\ ● & ● & ● \end{pmatrix}$$

by playing two H cards and one Z card.

Solution.— We start by playing an H card first, such that ◑ $\xrightarrow{H}$ ●, and then we apply a Z card on ◐, such that ◐ $\xrightarrow{Z}$ ◑. Finally, we play an H card again, such that ◑ $\xrightarrow{H}$ ●:

$$\begin{pmatrix} ◐ & ○ & ○ \\ ● & ● & ○ \\ ● & ● & ● \end{pmatrix} \xrightarrow[Z]{H} \begin{pmatrix} ◑ & ○ & ○ \\ ● & ● & ○ \\ ● & ● & ● \end{pmatrix} \xrightarrow{H} \begin{pmatrix} ● & ○ & ○ \\ ● & ● & ○ \\ ● & ● & ● \end{pmatrix}$$

Notice that the order of the operations, as long as one plays the H and Z cards in sequence, does not matter. That is, the proposed solution is equivalent to the following: ◑ $\xrightarrow{Z}$ ◐ $\xrightarrow{H}$ ● first and ◑ $\xrightarrow{H}$ ● second.

QM

Superposition. The kitten states ◐, ◑ are sometimes called *superposition states*. We have seen that they both give a uniform probability $1/2$ of resulting in either a ○, ● tile when measured. Does this mean ◐, ◑ are identical for the game purposes? Why would then one perform the ◐ $\xrightarrow{Z}$ ◑ operation? If there were no H card, the two ◐, ◑ tiles would be effectively equivalent and nothing in the game would make one preferable over the other. However, the H card can turn a ○ tile into a ◐ tile! Thus, if a player is playing White, their chances of getting a ○ tile in that square increase from $1/2$ to 1! Conversely, from ◑, one can first use the Z card to change it to ◐ and then with H achieve the desired ○ tile. Notice that one would not be able to do the same thing with a simple probability distribution, like a bag with mixed ○, ● tiles. These mixed bags do not transform under the operation H. Physicists call superpositions those states that can be changed into one another with certainty. This concept will be re-examined in greater depth along the whole book.

Two-squares operation cards

The only operation card that involves two game tiles is C_X. It, in fact, acts on two squares and can transform pairs of single-square states into entangled states, and vice versa.

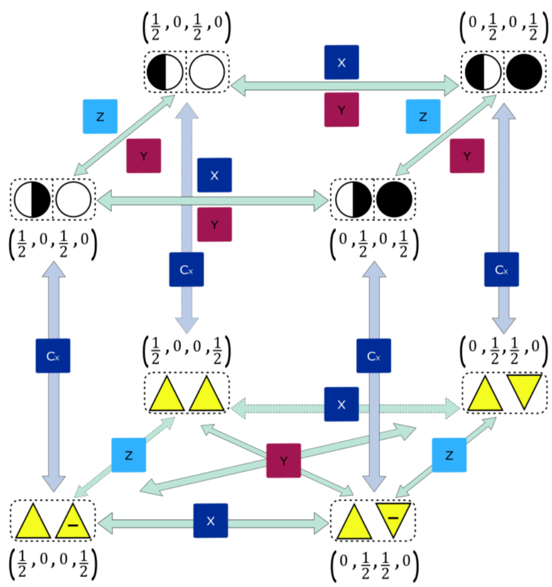


Figure 2.6: **Cubic map of double-square operations** X, Y, Z, C_X**.** The numbers in the parentheses (x, y, w, z), next to the symbols ◐○, ◐●, ◑●, ◑○ and △△, △▽, △▽, △△, represent the probabilities of obtaining ○○, ○●, ●○, ●● respectively, after performing a measurement, as explained in Sec. 2.3.3.

In Fig. 2.6 we see the action of the new C_X card that from pairs of states (on the upper face of the cube) represented by {◐○, ◐●, ◑●, ◑○}, map to *entangled* states (on the lower face of the cube), represented by {△△, △▽, △▽, △△}.

Note that the dashed rectangular box beneath the states on the top face of the cube is clearly divided by a dashed line, whereas the one beneath the states on the bottom face is not. This representation highlights the difference between pure separable states and pure entangled states of two qubits.

The C_X card can also be used on other combinations of circular tiles, and may involve more than two squares. This advanced use is investigated in [DeS25].

!

When applying a C_X card on a two-qubit state, the resulting state may differ depending on which of the two qubits — first or second from 1 to 9 on the game board — is designated as *control* qubit. Let us consider the state ◐○ and let us indicate with a letter C the control qubit. Then:

(C on first tile) ◐○ $\xrightarrow{C_X}$ △△

(C on second tile) ◐○ $\xrightarrow{C_X}$ ◐○

that is, when we apply C_X on the two qubit state ◐○, if we choose as control qubit the first one, we obtain an entangled state; if we choose as control qubit the second one, the C_X card acts as the I card.

If the starting state is an entangled state, applying C_X does not make a difference in terms of properties of the resulting state, in the sense that:

(C on first tile) △△ $\xrightarrow{C_X}$ ◐○

(C on second tile) △△ $\xrightarrow{C_X}$ ○◐

The two states ◐○ and ○◐ differ only in the reversed positions of the tiles. In a game scenario, one might be more advantageous than the other depending on the strategy used to maximize the chances of winning. In general, when applying a C_X card to an entangled state, it is not necessary to distinguish between control qubits: a player simply chooses how to arrange the tiles after the operation.

The C_X card acts:

- As the X card, when the control qubit has a ● tile on it; the other tile transforms as if X were applied to it.
- As in Fig. 2.6, when the control qubit has a ◐ or ◑ or a triangular tile on it; when applying it on entangled states you can choose the strategy for positioning the resulting tiles between two squares.

Exercise 2.3.2: Some C_X applications.

Apply the C_X card on states △▽, ●○, ●◑.
Solution.— We have:

$$△▽ \xrightarrow{C_X} ◑● \quad \text{or} \quad ●◑$$

according to the player's preference. Furthermore:

$$\overset{c}{●}○ \xrightarrow{C_X} ●●$$
$$●\overset{c}{○} \xrightarrow{C_X} ●○$$
$$\overset{c}{●}◑ \xrightarrow{C_X} ●◑$$
$$●\overset{c}{◑} \xrightarrow{C_X} △▽$$

As an example, consider Fig. 2.7, and imagine the black player has two C_X cards in the last turn of the game. Black will first play the C_X card on squares n. 1 and n. 6, resulting in the central game board of Fig. 2.7 and in an entangled state on those squares. The black player now plays C_X again and by choosing the control can either go back to the initial configuration or obtain ● on square n. 1 and ◑ on square n. 6. This is the most advantageous choice, as it allows to close the operational phase with a qtris in his favor.

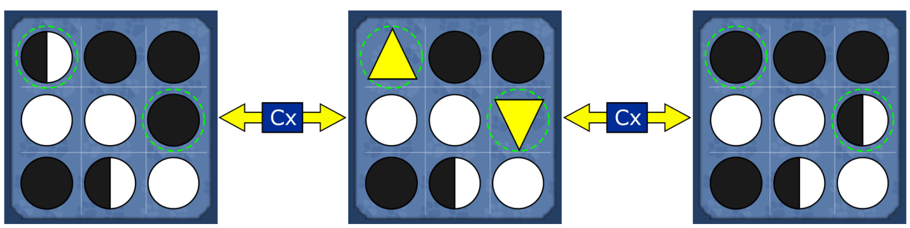


Figure 2.7: **Example of playing the C_X card.**

2.3.3 Measurement phase

At the end of the operations phase, the measurement phase begins. At the end of this phase, all tiles on the board will be either white or black. The procedure for measuring states on the board is shown in the following box.

Measurement of the game grid. 
Starting from square n. 1, and up to square n. 9:

- Check the type of tile present on the square.
  1. If it is a ○ or ● tile, move to the next square.
  2. If it is a ◑ or ◐ tile, roll a d100 die.

     If the result is between 1 and 50, replace the previous tile with a ○ tile.

     If the result is between 51 and 100, replace the previous tile with a ● tile.

     This is the meaning of $(1/2, 1/2)$ in the square map in Fig. 2.5.
  3. If it is a triangular tile, an entangled state is about to be measured.

     If a correlated pair is measured, that is, △△ or △△̄, roll a d100 die. If the result is between 1 and 50, replace the previous tiles with a ○○. If the result is between 51 and 100, replace the previous tiles with a ●●.

     If an anti-correlated pair is measured, that is, △▽ or △▽̄, roll a d100 die. If the result is between 1 and 50, replace the previous tiles with a ○●. If the result is between 51 and 100, replace the previous tiles with a ●○.

     This is the meaning of $(1/2, 0, 0, 1/2)$ and $(0, 1/2, 1/2, 0)$ in the cubic map in Fig. 2.6.

In quantum mechanics, to indicate that a measurement has been performed on a system, a meter symbol is used. In this rulebook, we will often use the following notation:

Starting state  $\longrightarrow$ Post-measurement state

Exercise 2.3.3: Measurement.

Consider a game board looking like this at the end of the operations phase:

$$\begin{pmatrix} \text{◐} & \text{◑} & \text{○} \\ \text{●} & \text{●} & \text{○} \\ \text{△} & \text{◑} & \text{▽} \end{pmatrix}$$

Perform the measurement.
Solution.— In order to measure the tiles on the game board, we roll a d100 die, starting from the first qubit up to the ninth.

1. On qubit n. 1, we roll a (54). Then ◐ $\longrightarrow$ ●.
2. On qubit n. 2, we roll a (71). Then ◑ $\longrightarrow$ ●.
3. On qubit n. 3, the tile is ○. Performing a measurement on the ○ tile in the {○, ●} basis always yields ○. We move to the next square.
4. On qubit n. 4, the tile is ●. Performing a measurement on the ● tile in the {○, ●} basis always yields ●. We move to the next square.
5. See point n. 4.
6. See point n. 3.
7. On qubit n. 7, we roll a (99). Performing a measurment on one qubit of an entangled state makes the entire state collapse. Then △▽ $\longrightarrow$ ●○.
8. On qubit n. 8, we roll a (24). Then ◐ $\longrightarrow$ ○

At the end of the measurement phase, the game board looks like:

$$\begin{pmatrix} \text{●} & \text{●} & \text{○} \\ \text{●} & \text{●} & \text{○} \\ \text{●} & \text{○} & \text{○} \end{pmatrix}$$

Both players made a qtris!

QM

Entangled states (1)**.** The behavior of the entangled states we showed upon measurement is that of strong correlations. This means that the outcome of one square completely determines the outcome of the other one. For example, the state △△ means that both tiles must have the same color, that is, perfect correlation, and which color it is is random with probability $1/2$. So, if the first one is black, the second one will be black. Perfect anti-correlations, like △▽, means that if the first is black the second is white and vice versa. Correlations are not just a quantum mechanical effect: they are also classical. You can easily see an example of anti-correlation if you hide in one hand a black tile and in the other a white tile. The probability of having either a black or white tile in one hand is $1/2$, but once you reveal it, the other one will be determined. So entangled states are correlated, but as we shall see, there is more to entanglement than just correlation.

QM

Measurement in quantum mechanics. In quantum mechanics, the unitary evolution of the state is deterministic. This means that, by using the rules for unitary operations, one can know exactly what state the system will be in at the end. However, the measurement does not give deterministic results, but only probabilities. In QTris, the result of a measurement is established by a roll of the die.
The method of calculating probabilities that allows to associate to a state the probabilities of providing certain observations is called *Born rule.* In the second part of the book, interested players and students can learn how to do these calculations as well. Note that measurement is not a unitary operation. This means that you cannot go back to the state before measurement: if the state after measurement is ○ you do not know if it is coming from ◑ or ◐.

2.3.4 Scoring

At the end of the measurement phase, each square of the board will have a white or black tile. At this point, it is possible to assign scores to the players. For each qtris made with its game symbol (white or black), the player gets one (1) point. Victory goes to the player with the highest score. Note that it is entirely possible that the two players score the same amount of points.

Exercise 2.3.4: Scoring 1.

Consider a game board looking like this at the end of the operations phase:

◐	◑	○
●	●	○
△	◐	△

Perform the measurement and assign scores.

Solution.— In this game, Player 1 plays Black and Player 2 plays White.

1. On qubit n. 1, we roll a (37). Then ◐ ⟶ ○.
2. On qubit n. 2, we roll a (3). Then ◑ ⟶ ○.
3. On qubits nn. 3, 4, 5, 6, we do not roll the die.
4. On qubit n. 7, we roll a (65). Then △△ ⟶ ●●.
5. On qubit n. 8, we roll a (18). Then ◐ ⟶ ○.

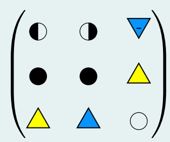
At the end of the measurement phase, the game board looks like:

○	○	○
●	●	○
●	○	●

Scoring	
Player 1	0
Player 2	1

Exercise 2.3.5: Scoring 2.*

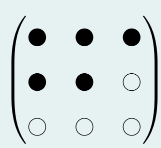
Consider a game board looking like this at the end of the operations phase:



Perform the measurement and assign scores.
Solution.— In this game, Player 1 plays White and Player 2 plays Black.

1. On qubit n. 1, we roll a 52. Then ◑ → ●.
2. On qubit n. 2, we roll a 89. Then ◐ → ●.
3. On qubit n. 3, we roll a 16. Then △▽ → ○●.
4. On qubits nn. $4, 5, 9$, we do not roll the die.
5. On qubit n. 6, we roll a 22. Then △△ → ○○.

At the end of the measurement phase, the game board looks like:



Scoring	
Player 1	1
Player 2	1

QM

Entangled states (2). From the previous leaflet of QM and the understanding of the QTris rules about entangled states, we have found that entanglement shows correlations. We also mentioned earlier that there is more to entanglement than just correlations. Let us consider again an example of classical correlations. Think of two boxes: one contains two white balls and one contains two black balls, but you do not know which is which. You open the box and draw a ball - if it is white (black), you know that both balls were white (black); you have 50% probability of finding a white box and 50% probability of finding a black box. Now, think of the entangled state △△: if upon measurement on the first qubit you obtain ○ (●), you know that you need to place a ○ (●) tile on the second qubit. What is the core difference between these two examples? The △△ state can be "undone" by applying the unitary operation C_X, together with H on the first qubit, into the starting state ○○. We do not need to learn the measurement outcome to return to the unentangled state. The classical box-mixture cannot be deterministically undone. That is, once you randomly pick white-box or black-box, there is no way to undo that choice without learning which box you actually have by extracting a ball: there is no reversible operation that restores the case *two white balls* (*two black balls*) without first finding out which box you have.

Exercise 2.3.6: Deterministic undoing of entangled states.

Consider quantum states △▽, △△(−), △▽(−). Prove that we can go back to states ○●, ●○, ●● with some unitary operations.
Solution.— We have:

△▽ $\xrightarrow{C_X}$ ◐● $\xrightarrow{H_1}$ ○●

△△(−) $\xrightarrow{C_X}$ ◑○ $\xrightarrow{H_1}$ ●○

△▽(−) $\xrightarrow{C_X}$ ◑● $\xrightarrow{H_1}$ ●●

The subscript 1 indicates that the H card is played on the first qubit, namely the square with the black-and-white tiles above.

Summary of phases and game actions.

1. Preparation phase
   - (a) Fill the entire game grid with valid states.
   - (b) Distribute four (4) cards to each player.
   - (c) Establish the order of play (roll of the die).
   - (d) Mulligan.
2. Operations phase
   - Players take turns, following the previously established order of play. The number of turns for each player is five (5).
   - Structure of each game turn:
     - (a) Draw two (2) cards from the operations card deck.
     - (b) Play two (2) operations cards from the hand.
3. Measurement phase
   - Carry out the measurement procedure of the game grid as reported in the box in Sec. 3.3.1.
4. Scoring phase
   - For each qtris made with their game symbol, players receive one (1) point. The player with the most points wins the game.

Chapter 3

Advanced Rules

In this chapter, we lay down advanced rules to play QTris. The most important of these rules is the one that allows for more interesting probabilities than those seen so far. The other rules take into account deeper aspects of quantum mechanics and make the game depth much greater and challenging. We start by first introducing more quantum mechanical effects that can be explained within QTris, and then adding new rules in QTris, to deal with even more quantum mechanics. Some of the rules can be just added to the pre-existing rules of the previous chapter; in other examples, we introduce game variants, for instance, starting with different preparations or using a different turn system.

> **!** A general remark about QTris rules is about the usage of cards on states that are not covered by the card rules. One can ask: «Why cannot I use the C_X card on two qubits, one of each already belongs to an entangled pair?» Well, in QM, in principle, every unitary operation can be used on every state *if one has the experimental capabilities for doing so*. In practice, many times we can only use some operations in some circumatances. This is reflected in the lack of some rules for using some cards on same states in QTris. However, in principle one could. This corresponds to advancing the technology. QTris shows how to systematically make up new rules about how to use old cards on new states or new cards altogether. This very much is the kind of problems that theorists and experimentalists in quantum information face in their research activity.

3.1 More quantum mechanics in QTris*

As we have said before, there is nothing in quantum mechanics that cannot be performed in QTris. There are virtually infinite additional games that can be played on QTris that have a counterpart in quantum mechanics, and quantum experiments that can be demonstrated on QTris. In this section, we will explore some of these issues!

The two main facts of quantum mechanics we want to investigate are the difference between pure and mixed states and what goes on in a measurement.

One question often asked in quantum mechanics is: «What happens to a quantum state after a measurement?» Well, as we have seen, so far measuring meant observing whether a tile was white or black. We have seen that when a tile is measured, the response will be either white or black with a certain probability (determined by the roll of a die). What happens next? If nothing else intervenes, the tile remains white or black! This seems to be a very natural property! Quantum physicists call it collapse and this is the topic of the next leaflet.

QM

Collapse. Quantum physicists often talk about collapse do describe what happens to a state after measurement. What does this mean? Consider the case of the quantum state ◑. We know that the probability of finding the white tile is $1/2$. At this point, the tile is actually white! Now, if you asked: «What is the probability that I will observe a white tile from now on?» Well, if nothing happens in the meantime (i.e., if no other operation intervenes), the tile will definitely be white! This is just a way of saying that physical properties are stable. The state must therefore change from a ◑ tile to a ○ tile. That is all: perhaps talking about *collapse* is a bit melodramatic.

At this point, one might ask: «Can I only observe the color of a tile? Or are there other measurements I can make?» Can one measure, for example, mass, energy, the *fluffiness*! In particular, in quantum mechanics, there are some odd properties: like that of a black-and-white tile that can be oriented to the right or to the left (remember, for single-color tiles orientation does not matter). If a tile is white, can we observe its orientation? Of course! We can observe the property of being ◑ or ◐; these will manifest with a probability equal to those in Fig. 2.6! See the next QM leaflet.

We have learned something very interesting. We can measure the property of *color* or the property of *orientation*. An interesting game could be this. Let's prepare a white tile ○, and then perform a series of sequential measurements. For example, we first measure the orientation, then the color, and then the orientation again. What will happen? Well, with a probability of $1/2$, the tile will be oriented to the left ◐, and with the same probability to the right ◑. Suppose we obtain a ◐ tile. If we now measure the color, we know we can obtain with the same probability of $1/2$ either a white ○ or a black ● tile (without any particular orientation). We now well understand what might happen if we measure the orientation again: we will obtain either ◐ or ◑ with a probability of $1/2$. This simple game turns out to be one of the fundamental experiments of quantum mechanics, as we will see in the note below!

QM

Measurement in different bases. Here's an interesting question: «Is there a physical property corresponding to being ◐ and another property corresponding to being or ◑, so that these properties are mutually exclusive?»
Two properties are mutually exclusive if, when one is certain (i.e., has probability 1), the other is impossible (i.e., has probability 0). Now, if these properties physically exist, it means we should be able to design an experiment to detect them! Let's remember that physics is an experimental science — it is ultimately concerned with things that can be observed.
To perform this experiment, we need to build a device that, when the state is ◐, gives the result ◐ with probability 1, and so on.
Here is how to build such a device: we build an apparatus that first applies the H operation and then measures the color property. Let's see how this works.
If the state is ◐, after applying H, it becomes ○, and a color measurement will give white with probability 1 (and vice versa for ◑ $\overset{H}{\longleftrightarrow}$ ●).
At this point, all we need to do is change the labels on the apparatus's indicator lights: replace *white* with ◐ and *black* with ◑. That's it —– the experiment is ready!
In QM, measuring a new physical property is called measuring in a different basis associated to that property. Notice that with respect to the orientation basis ◐, ◑ the solid ○, ● behave like Schrödinger's kittens! Being a Schrödinger's kitten is a property relative to a basis.

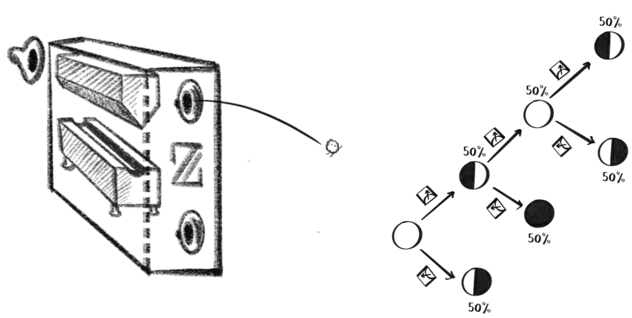


Figure 3.1: **The Stern-Gerlach experiment.**

3.1.1 The Stern-Gerlach experiment

QM

The Stern-Gerlach experiment. This experiment uses silver atoms, which we can imagine behaving like tiny magnets when subjected to a magnetic field. The experiment begins by heating silver atoms in a furnace. The thermally agitated atoms exit the furnace through a small hole, forming a beam, and are directed towards a non-uniform magnetic field using a device called a collimator. After passing through the magnetic field, the silver atoms strike a screen capable of recording their impact points.

The experiment shows that the silver atoms are deflected either upward or downward, striking only two narrow regions of the screen. The Stern-Gerlach apparatus measures a physical property of the electron and other particles called *spin*, along a given direction: the spin of the atoms in this experiment takes on only two discrete values — just like the color of the little tiles in QTris.

Suppose we use a Stern-Gerlach apparatus aligned along a z direction. Further, let's suppose we select only the atoms that are deflected upward along z. If we pass these atoms through another Stern-Gerlach apparatus aligned along z, we observe that they are always deflected upward. This tells us that spin along z is a stable physical property.

Now, let's imagine passing these same atoms through a Stern-Gerlach apparatus aligned along a x direction, perpendicular to z. We observe that the atoms are now deflected either to the right or the left. If we select only one of these two branches and pass those atoms through another Stern-Gerlach apparatus aligned along z, we once again see that the atoms are deflected either upward or downward.

This tells us that spin along z and spin along x are incompatible physical properties — just like the color and orientation of the tiles in QTris!

The Stern-Gerlach experiment described above is something that can be played with on a single square of QTris, as you may have realized. Before becoming quantum physicists, regular physicists were quite perplexed because it seemed inconceivable to them that two properties of the silver atoms in the experiment could not be determined simultaneously. That was until Heisenberg came along and established a principle stating that this indeterminacy is fundamentally at the core of quantum mechanics. Since then, physicists have become quantum physicists.

QM

Heisenberg's Uncertainty Principle. We have seen that qubits may have different binary properties, for instance, color and L-R orientation. Both color and orientation come into two mutually exclusive outcomes: (○, ●) for color and (◐, ◑) for orientation. So far so good. Now consider another example, the Knight chess piece. This one ♞ is at the same time black *and* left. Compare with the qubit symbol that is black ●, you have no idea whether it is left or right! You see that the Knight pieces have four possibilities: ♞, ♘, ♘, ♞ and compare these four states with the four qubit states ○, ●, ◐, ◑! We see that while color and orientation are compatible properties for the Knights - that is, you can tell exactly where they are facing and what color they have - for the qubits having a definite color means not having a definite orientation and vice versa! Indeed, we know that the state, say, ◐, has only $1/2$ probability of being white, while - measuring in the orientation basis - the ● has only $1/2$ probability of being left.
Quantum physicists say that these two properties —— color and orientation (for the qubits) —— are *incompatible*. As a result, if one property is determined, the other is undetermined. And there you have it: *Heisenberg's uncertainty principle*.

QM

The ground beneath her feet. Very often the general public — and some physicists — get queasy about all this indeterminacy; it seems that one is losing the ground beneath one's feet. In his novel *The Ground beneath her Feet* from 1999, Salman Rushdie wrote:

> «In an age of great uncertainties it is easy to mistake science for banality, to believe that Heisenberg is merely saying, gee, guys, we just can't be sure of anything, it's all so darn uncertain, but isn't that, like, beautiful? Whereas he's actually telling us the exact opposite: that if you know what you're doing you can pin down the exact quantum of uncertainty in any experiment, any process. To knowledge and mystery we can now ascribe percentage points. A principle of uncertainty is also a measure of certainty. It's not a lament about shifting sands but a gauge of the solidity of the ground.»

One simply cannot put it more clearly and better. In the second part of the book, you will learn how to *compute* uncertainty *exactly*.

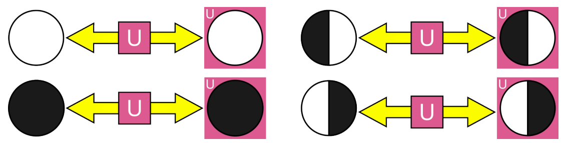


Figure 3.2: **Effect of the U operation card on single-square tiles.** After applying the U operation card to a white, black or black-and-white tile, you must decorate it by placing a pink tile underneath. Conversely, if you apply the U operation card again, you must remove the pink tile. This decoration rule is also valid when using a U card on any tile of an entangled state, see Fig. 3.5.

3.2 The U card and non-flat probabilities*

So far, we have seen that we can transform our states and change probabilities from some certainty to absolute uncertainty. For example, on one tile one can go from ○ to ◑, that is, from $(1, 0)$ to $(1/2, 1/2)$. On entangled states, we can obtain probabilities like $(1/2, 0, 1/2, 0)$, see Fig. 2.6. The game might become more interesting if other probabilities might be attained, say $(3/4, 1/4)$. To this end, we introduce a new card U.

The action of this card on the kittens is readily given in Fig. 3.2. As you can see, it is very simple, it just decorates the tiles with the pink card U: ○ $\overset{U}{\leftrightarrow}$  and ● $\overset{U}{\leftrightarrow}$ .

How are these new tiles transformed one into another? We just adopt the map of single-square operations as in Fig. 2.5. However, now we have different probabilities for the outcomes ○, ●. The new map with the operations and the final probabilities is shown in Fig. 3.3. [1]

> QM
>
> **Magic in QM.** In quantum mechanics, all the operations described until now are called *Clifford operations*. The very interesting thing is that most of the features of quantum mechanics that allow for important applications like quantum computing and quantum sensing or communication do rely on operations outside the Clifford ones. This means that without other resources, no quantum computer or sensor can obtain any *quantum advantage* compared to a classical one. The quantum resource missing is colloquially called *magic* [OLHL22]. If one has access to a bit of magic then quantum advantage can be unlocked. And behold, the unitary transformation U is the one that injects magic in a quantum system!

[1] The reader who is familiar with all the rules of QM will have understood that X is the bit flip and that in this new diagram it is the bit flip in the new basis.

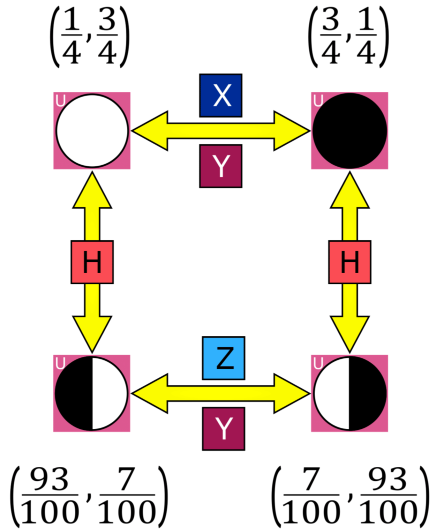


Figure 3.3: **Map of single-square operations X, Y, Z, H on the decorated U tiles.** The numbers in the parentheses (x, y) next to the symbols ○, ●, ◐, ◑ represent the probabilities of obtaining ○ and ● respectively, after performing a measurement, as explained in Exercise 3.2.2.

Exercise 3.2.1: Clifford operations.

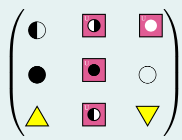
Show that if one only uses the cards I, X, Y, Z, H, C_X one can only obtain flat probabilities, that is, $(1, 0, 0, 0)$ or $(1/2, 1/2, 0, 0)$, $(1/4, 1/4, 1/4, 1/4)$ and the like.
Solution.— Let us define the following sets of game tiles:

S_1 := {○○, ○●, ●○, ●●}
S_2 := {◐○, ◐●, ◑○, ◑●}
S_3 := {△△, △▽, △△, △▽}
S_4 := {△△, △▽, △△, △▽}

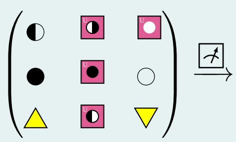
That is, we do not consider the U tiles. As shown in figures Fig. 2.5, Fig. 2.6 and Fig. 3.4, when we act with the I, X, Y, Z, H, C_X cards on such tiles, regardless of which of the operations is applied, the resulting states always lie within the S_1, S_2, S_3, S_4 sets. Clifford operations — in QTris I, X, Y, Z, H, C_X — possess the property of sending the states of S_1, S_2, S_3, S_4 into states of S_1, S_2, S_3, S_4, according to the rules of the game, while remaining entirely within them. This property is perfectly encapsulated by the rectangular enclosure in the diagrams, which also shows that all possible probability distributions are flat.

Exercise 3.2.2: Measurement.

Consider a game board looking like this at the end of the operations phase:



Perform the measurement.
Solution.— To perform the measurement, we look at the probabilities for every square. We have, as probabilities of obtaining white:



$$\begin{pmatrix} 50\% & 7\% & 25\% \\ 0 & 75\% & 1 \\ 50\% & 93\% & \text{opposite of square 7} \end{pmatrix}$$

Notice that the probability of obtaining white on square n. 7 is 50%. Once the square n. 7 is determined, the square n. 9 will be the opposite: it will also come with a probability that is 50%, but it is not independent. It is perfectly anti-correlated with that of square n. 7.

To perform a measurement on the pink decorated tiles, we roll a d100 and we follow the probabilities in Fig. 3.3 for the die, as seen in Exercise 3.2.2.

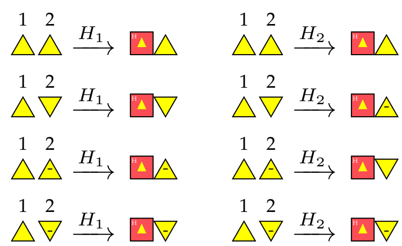
QM

Partial incompatibility. Consider the operation U as in QTris. The operation U has defined a new pair of properties , . These two properties are mutually exclusive, of course. Let us call this property $U-$ness. Our question is: «Is $U-$ness compatible or not with, say, color?» We know that, for instance, orientation is incompatible with color. What about $U-$ness? Let us see.
Let us pick a state with definite $U-$ness, say . We know that upon measurement, it will return a probability $1/4$ of being ○. As we can see, determining $U-$ness still implies some uncertainty on color. However, this time the uncertainty is less, as it is much more likely for the system to be black $(3/4)$ than to be white $(1/4)$. In the case of the state  (that we call $U-$left) instead, the probability of white is $93/100$, see Fig. 3.3. This means that  is almost compatible with being white, but not quite. In QM we can also have partial incompatibility. Partial incompatibility is at the root of quantum non-locality [AH25].

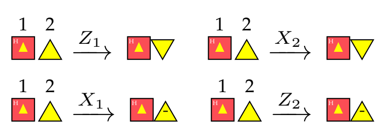
3.3 Entanglement and magic *together**

In this section, we show the rules about how C_X acts together with the H and U cards. As we know, the C_X card creates entanglement when acting on the {◐○, ◐●, ◑○, ◑●} tiles, introducing in QTris new states {△△, △▽, △△̄, △▽̄}. The natural question that arises is: «As we do for the other tiles of the game, can we apply the H and U cards to the states obtained after playing C_X?» The answer is, *yes*!

Let us take a look at Fig. 3.4. We have: {△△, △▽, △△̄, △▽̄} $\xrightarrow{H}$ {■△, ■▽, ■△̄, ■▽̄}. First, we observe that the H card acts on the first and the second qubit of the entangled states in the same way, up to an unobservable global phase. That is,



where 1 denotes the first qubit and 2 denotes the second qubit, namely, the square occupied by the first and the second tile. In both cases, the resulting states have probability $(1/4, 1/4, 1/4, 1/4)$ of being {○○, ○●, ●○, ●●}. That is, by playing the H card, the probabilities remain flat, but now each state has identical probability. Second, we observe that the X, Y, Z cards act on the {■△, ■▽, ■△̄, ■▽̄} tiles almost the same way as they act on the entangled states without applying H. The only difference is that we can use both the X card and the Z card to obtain a certain state, since one operation acts on the first qubit and the other on the second qubit. For example:



During the game, it is not necessary to specify on which qubit we apply the X and Z cards: just be aware that both can be used.

The U card acts on entangled states {△△, △▽, △△̄, △▽̄} in a manner entirely analogous to the H card. The key difference is that, as with the single-square states decorated with U, it introduces new probabilities such that the resulting decorated states {■△, ■▽, ■△̄, ■▽̄}, while preserving all the properties we have analyzed, exhibit a non-flat probability distribution of being {○○, ○●, ●○, ●●}. Such probability distribution is shown in Fig. 3.5.

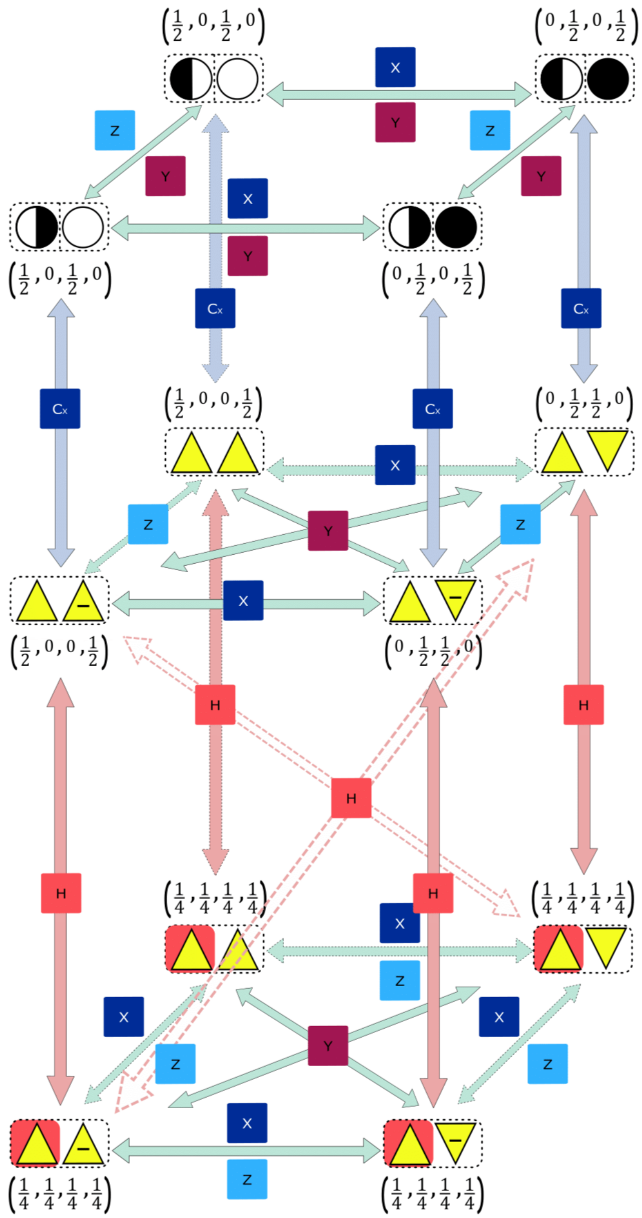


Figure 3.4: **Map of all the operations allowed by using entangled states together with H and the corresponding probabilities for the outcomes {○○, ○●, ●○, ●●}.**

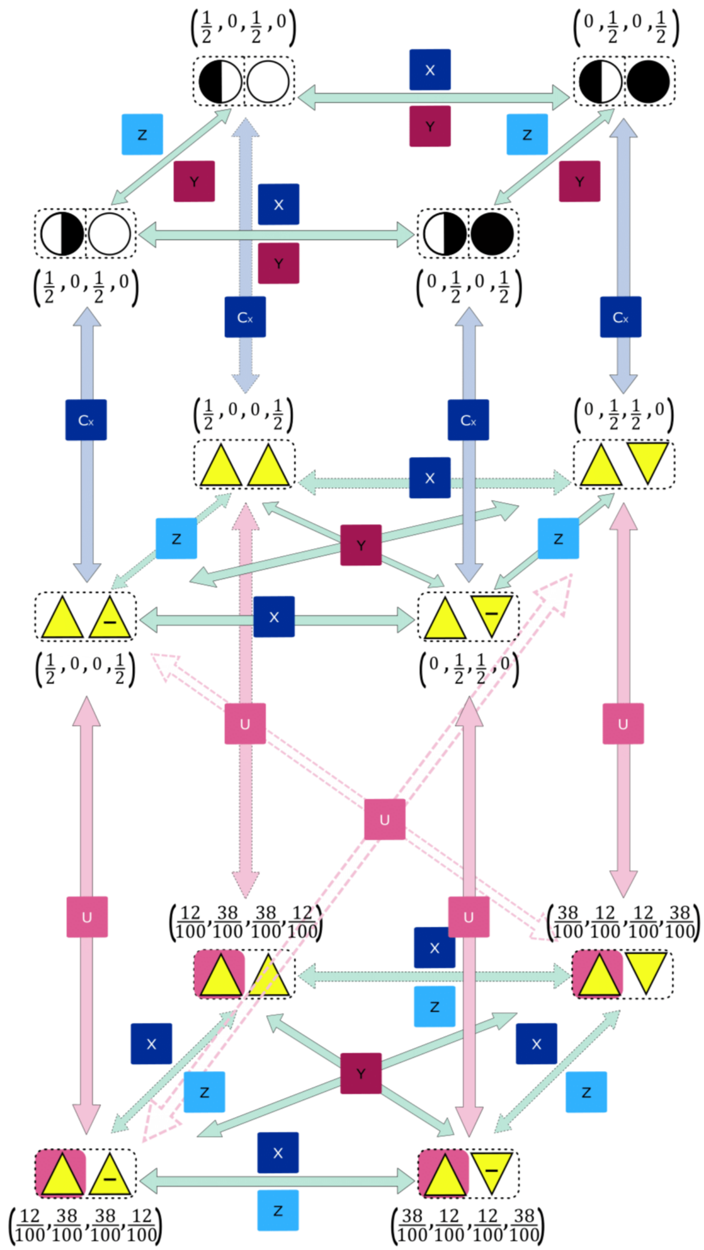


Figure 3.5: **Map of all the operations allowed by using entangled states together with U and the corresponding probabilities for the outcomes {○○, ○●, ●○, ●●}.**

3.3.1 Advanced measurement

Measurement of the game grid.
Starting from square n. 1, and up to square n. 9, check the type of tile present on the square.

- If the tile on the square is a triangular tile decorated with a red tile, roll a d100 die.

  1. If the result is between 1 and 25, replace the previous tiles with a ○○. If the result is between 76 and 100, replace the previous tiles with a ●●.
  2. If the result is between 26 and 50, replace the previous tiles with a ○●. If the result is between 51 and 75, replace the previous tiles with a ●○.

- If the tile on the square is a triangular tile decorated with a pink tile, we measure the entangled state as follows.

  If a correlated pair is measured, that is, or , roll a d100 die.

  1. If the result is between 1 and 12, replace the previous tiles with a ○○. If the result is between 89 and 100, replace the previous tiles with a ●●.
  2. If the result is between 13 and 50, replace the previous tiles with a ○●. If the result is between 51 and 88, replace the previous tiles with a ●○.

  If an anti-correlated pair is measured, that is, or , roll a d100 die.

  1. If the result is between 1 and 38, replace the previous tiles with a ○○. If the result is between 63 and 100, replace the previous tiles with a ●●.
  2. If the result is between 39 and 50, replace the previous tiles with a ○●. If the result is between 51 and 62, replace the previous tiles with a ●○.

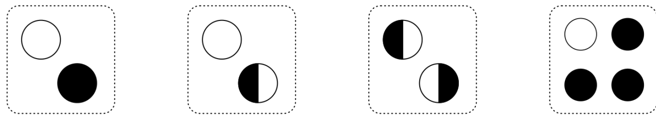


Figure 3.6: **Mixed states.** Four examples of mixed states. Both ⚁ and ⚁ are completely mixed states and do not change under any card. They also give the same probabilities $(1/2, 1/2)$. The state ⚁ is not completely mixed and gives probabilities $(3/4, 1/4)$. It has limited ways of transforming with the cards. The state ⚃ is mixed with probabilities $(1/4, 3/4)$.

3.4 Mixed states*

In quantum mechanics, in many realistic scenarios — such as systems with incomplete information, statistical ensembles, or interactions with an environment — a system cannot be described by pure states, aka kittens. In QTris, for example, performing a measurement results in a statistical mixture of states ○, ●, which we obtain with probability $1/2$. In such circumstances, the system is described by the so-called *mixed states*, which we introduce in this section. Let us introduce a notation for mixed states. For example, if we place on one square a ○ tile together with a ● tile, we say that this is the state with probability $1/2$ of being either ○, ●, and we denote it with ⚁. If we place on one square one tile ○ and three tiles ●, this is the state of probabilities $(1/4, 3/4)$ for (○, ●), while if we place three tiles ○ and one tile ●, this is the state of probabilities $(3/4, 1/4)$; we denote them, respectively, with ⚃, ⚃. To construct a mixed state, in general, one must place more than one tile on a single square, and which tiles are placed and how many of each determine the probability (x, y) of the mixed state being (○, ●). Fig. 3.6 shows several examples of mixed states. Some mixed states have uniform probabilities, like the states ⚁, ⚁ in Fig. 3.6: they are called *completely mixed states*.

3.4.1 Mixed states vs kittens

In QTris, we know that the states ◐, ◑ give probabilities $(1/2, 1/2)$ of obtaining (○, ●). So far so good. We have also found that the state ⚁, with two tiles on one square, yields ○ or ● with equal probabilities $(1/2, 1/2)$. So one could say: «What is the big deal about these kittens ◐, ◑? They just represent even probabilities for ○, ●». As we have mentioned before, there is a huge difference between mixed states, that represent just probabilities for some outcomes, and pure states, the so-called kittens, in quantum mechanics. The rules of QTris show us that kittens have, in addition to providing us with probabilities, many other properties. We now want to explain what is this difference in

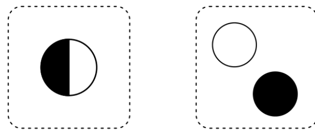


Figure 3.7: **Superpositions vs mixed states.** The state on the left is *pure*, that is, it is a kitten. There is a deterministic way of transforming it in any other pure state. On the right hand side, there is a (completely) mixed state, and there is not a deterministic way of turning it in a state giving the desired probabilities.

detail. Those of you who have read some popular science books in quantum mechanics, will understand that we are talking about the difference between mixtures and *quantum superpositions*, see Fig. 3.7.

Let us then see what is the difference between, say, the pure state ◐ and the mixed state . They both return the same probability $1/2$ of obtaining either ○ or ●. However, they behave very differently when playing the operation cards. If we play the H card on ◐, we obtain ○ and we have a 100% probability of getting ○ upon measurement. Similarly, by playing Z first and then H we obtain ● with 100% probability. If we play the H card on the *mixed state* , we have ○ $\xrightarrow{H}$ ◐ on the first tile, and ● $\xrightarrow{H}$ ◑ on the second tile. The new state on the square is . Now, what probabilities will these tiles return? The tile ◐ comes with probability $1/2$ and returns probability $1/2$ for both ○, ●, and the same holds for ◑. So in the end, we get a probability $1/2$ for ○, ●, exactly as in the initial state. We then see that the card H is completely useless on the mixed state , but it has a fundamental function for the state ◐.

Another example is the following: the states , have probabilities $(1/4, 3/4)$ and $(3/4, 1/4)$ of begin (○, ●), but they are pure, while the states and have probabilities $(1/4, 3/4)$ and $(3/4, 1/4)$, and they are mixed (not completely).

Exercise 3.4.1: Completely mixed states are useless.

Show that no card is useful on the completely mixed state [○●]. Similarly, show that this is also true for the mixed state [◐◑].
Solution.— We show the action of each game card on the [○●], [◐◑] states and the probability of the resulting states.

$$[○●] \xrightarrow{X} [●○] \qquad p([●○]) = (1/2, 1/2)$$
$$[○●] \xrightarrow{Y} [●○] \qquad p([●○]) = (1/2, 1/2)$$
$$[○●] \xrightarrow{Z} [○●] \qquad p([○●]) = (1/2, 1/2)$$
$$[○●] \xrightarrow{H} [◐◑] \qquad p([◐◑]) = (1/2, 1/2)$$
$$[◐◑] \xrightarrow{X} [◐◑] \qquad p([◐◑]) = (1/2, 1/2)$$
$$[◐◑] \xrightarrow{Y} [◑◐] \qquad p([◑◐]) = (1/2, 1/2)$$
$$[◐◑] \xrightarrow{Z} [◑◐] \qquad p([◑◐]) = (1/2, 1/2)$$
$$[◐◑] \xrightarrow{H} [○●] \qquad p([○●]) = (1/2, 1/2)$$

Exercise 3.4.2: Not completely mixed states have some use.

Consider the state with three tiles ● and one tile ○ on one square, that is, [○●●●]. Show what happens if one uses the card X on it.
Solution.— We start with the state [○●●●] of probabilities $(1/4, 3/4)$. By using X, all the three ● flip as ● $\xrightarrow{X}$ ○, while the one ○ flips as ○ $\xrightarrow{X}$ ●, thus resulting in the mixed state [○○○●] with three ○ and one ●, with probabilities $(3/4, 1/4)$. As you can see, not completely mixed states can be partially manipulated, but not into something in which the probabilities become certainties.

QM

So, what are Schrödinger's kittens? Recall our previous QM leaflet called *Superposition* at page 21. We have seen that Schrödinger's kittens ◐, ◑ have the property of not having a well-defined color until measured, after which the color is established with a probability of 1/2. However, there exist unitary operations that take them to the state with a chosen color with certainty. Schrödinger's kittens thus have this dual nature: if one observes them, they are white or black at 50%, but there is a way to make them definitely white or black! Quantum physicists say that this kind of property is that of a *coherent superposition*, but these are just fancy words to say the same thing we have explained in terms of the H and Z operations.

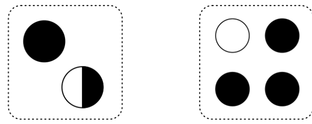


Figure 3.8: **Two different mixed states.** States ⚁ and ⚃ have the same probability $(1/4, 3/4)$ of being (○, ●), but they are very different.

3.4.2 Mixed states vs mixed states

We have seen that the mixed state ⚃ has probability $(1/4, 3/4)$ of being (○, ●). The mixed state ⚁, consisting of one ● tile and one ◑ tile, also has probability $(1/4, 3/4)$ of being (○, ●). By now, we are familiar with quantum mechanics, and we know that these two states, shown in Fig. 3.8, cannot simply be identical. A detailed discussion of all the differences between mixed states is deferred to the treatment in [DeS25]. However, we can already introduce some exercises.

Exercise 3.4.3: Measuring mixed states in a different basis.**

Measure the two mixed states ⚁ and ⚃ in the {◐, ◑} basis and prove that one obtains different probability outcomes.
Solution.— Let us start from the mixed state ⚁. The tile ● has probability $1/2$ of being either ◐ or ◑. The tile ◑ has probability 1. Then the probability of the mixed state ⚁ of being (◐, ◑) is $(1/4, 3/4)$.
Now, let us consider the mixed state ⚃. The tile ○ has probability $1/2$ of being either ◐ or ◑. Each of the three tiles ● also has probability $1/2$ of being ◐ or ◑. Then the probability of the mixed state ⚃ of being (◐, ◑) is $(1/2, 1/2)$.

Exercise 3.4.4: Mixing with kittens is better.**

Show that the state can be manipulated in a way to obtain better winning probabilities for the white than using .

Solution.— As we have seen, from we can obtain the state giving a probability for the white of $3/4$. Now, let us start with and let us see if we can beat it. If we apply U, we obtain with both tiles decorated by the pink U tile. So far so good. Now apply the operation X. The tile will flip into , while the tile stays put. So we have probability $1/2$ of having with the white winning with probability $3/4$, and probability $1/2$ of having with probability for the white of $93/100$ resulting in $1/2(0.75 + 0.93) = 0.84$, which is way better than $3/4$!

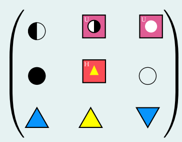
3.4.3 Measurement and mixed states

At this point, the attentive player may have noticed that, after a measurement, one has just several outcomes of ○, ● with certain probabilities on every tile, that is, a mixed state on every tile! Measurement then produces mixed states (unless one is in one of the certainty states ○, ● to start with). We can play with this by adding a rule to make mixed states part of the game of QTris. The new rule of QTris to simulate noise in the game is the following.

After every turn, one random tile is measured. One of the players rolls a d10 die, and with the outcome of $1 - 9$, the corresponding tile gets measured. With an outcome of 10, nothing happens. If the tile has a well-determined color ○, ●, they stay quite the same. However, if the tile is one of the states ◑ or ◐, we know that upon measurement we obtain one of the ○, ● with 50% of probability. In this case, as we have seen, we place on that tile *both* the tiles ○, ● or ◐, ◑ to denote the completely mixed state. Similarly, if there were decorated states like the kittens , we put the appropriate number of ○, ● for the mixed state of the corresponding probabilities.

Exercise 3.4.5: Playing with mixed states in QTris.*

In this game variant, we start with a more complicated preparation and we use measurement as part of the operations the players can perform. The players start with four (4) cards each and play two (2) cards each per turn, then perform a measurement and draw two (2) cards from the deck. To perform the measurement, the players roll a d10 and upon the outcome, the corresponding square gets measured, resulting often in a mixed state. On a outcome of 10, no square gets measured at all. In this game we play a total of four (4) turns.
Consider a game board looking like this at the end of the preparation phase:



Player 1 plays as White and Player 2 plays as Black.
Player 1 has cards: I, X, H, U. Player 2 has cards: Y, Z, C_X, U.
Solution.— The operations phase starts.

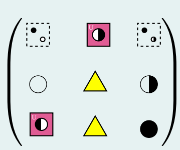
1. **Turn** 1. Player 1 plays X on qubit n. 4: ● $\xrightarrow{X}$ ○ and H on qubit n. 5: [▲]△ $\xrightarrow{H}$ △△. The turn ends. Player 1 rolls a d10: [⬡1]. On qubit n. 1, we have ◐ $\xrightarrow{[\text{measure}]}$ [⁛]. Player 1 draws Z, H.

2. **Turn** 2. Player 2 plays Y on qubit n. 6: ○ $\xrightarrow{Y}$ ● and C_X on qubit n. 7: ▲▼ $\xrightarrow{C_X}$ ◐●. The turn ends. Player 2 rolls a d10: [⬡4]. On qubit n. 4, we have ○ $\xrightarrow{[\text{measure}]}$ ○. Player 2 draws I, X.

3. **Turn** 3. Player 1 plays H on qubit n. 6: ● $\xrightarrow{H}$ ◑ and U on qubit n. 7: ◐ $\xrightarrow{U}$ [◐]. The turn ends. Player 1 rolls a d10: [⬡10]. Nothing happens.

4. **Turn** 4. Player 2 plays I and Z on qubit n. 7: [◐] $\xrightarrow{Z}$ [◑]. The turn ends. Player 2 rolls a d10: [⬡3]. On qubit n. 3, we have [○] $\xrightarrow{[\text{measure}]}$ [⁛].

At the end of the operations phase, the game board looks like this, with probability of obtaining white:



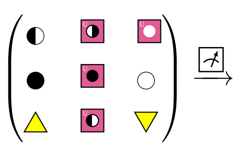
$$\xrightarrow{[\text{measure}]} \begin{pmatrix} 50\% & 7\% & 25\% \\ 1 & 50\% & 50\% \\ 93\% & 50\% & 0 \end{pmatrix}$$

QM

Non-unitary evolutions and mixed states. One important element of QM is that evolution is not always unitary, so it does not necessarily resembles one of the operations the players can perform and that have been described by the cards so far. In the context of quantum computation, these non-unitary operations are sometimes referred to as *noisy* operations. Non-unitary (i.e., noisy) operations do not keep pure kittens pure but make them mixed. We have seen that measurement is one such noisy operation. There are more noisy operations other than measurement and they are described by an elegant formalism that will be explained in the second part of the book.

3.5 Statistics in QTris*

As we have seen, at the core of QTris (and QM) there is the fact that after all the operations have been performed, there are probabilities of obtaining ○, ● on each square. For example, from Exercise 3.2.2, we see that the following game grid will feature these probabilities for the white tile:



$$\begin{pmatrix} 50\% & 7\% & 25\% \\ 0 & 75\% & 1 \\ 50\% & 93\% & \text{opposite of square 7} \end{pmatrix}$$

Then, by rolling the dice, we have a possible outcome, say

$$\begin{pmatrix} \circ & \bullet & \circ \\ \bullet & \bullet & \circ \\ \bullet & \circ & \circ \end{pmatrix}$$

resulting in a score $1 - 0$ for the white.

What would happen if we repeated the measurement many times starting from the state after the operations? The laws of probability tell us that if you repeat it many times, the statistics of the results will be very close to that of the probabilities! This means that on average half of the times the outcome of the first square will be white and half of the time black, while on the second square the white will appear only a fraction $7/100$ of the times. You can play and roll the dices many times and keep track of the results and check. What about the scores? Well, the average score is not the one given by the average result of the table. Rather, one has to score for every outcome and *only after* average all the scores. Try! A more agile way of doing all this is to put it in a computer, or, to play with the QTris app that is coming out soon!

3.6 The third kitten ($|i\rangle$ basis)*

In QM, a basis is a set of states in which you choose to describe or measure your system. In QTris, as we know, the basis in which the system is measured after performing all operations is the one defined by the game tiles {○, ●}. We introduce a new basis, consisting of tiles {◓, ◒}: they describe the properties *up* or *down* (U-D orientation), which are mutually exclusive. Notice that the basis {○, ●} is left unchanged by the operation Z, while X and Y move within the basis; the basis {◐, ◑} is left unchanged by the operation X, and one uses Z and Y to move within the basis; to move between the two bases, we play an H card. The new basis {◓, ◒} is the one left unchanged by the operation Y. To transform ◓ into ◒, and vice versa, we either apply the X card or the Y card. The new S card acts as a change between bases. The complete action of the new game components together with those already present in QTris is shown in Fig. 3.9. It is possible to play the extended advanced variant using the new cards and the new tiles.

Card	S Expansion
I	5
X	10
Y	5
Z	10
H	12
C_X	10
U	9
S	7

The use of the U and C_X cards on the new tiles is explored in [DeS25]. Therefore, to play this version, it is sufficient to follow the schemes illustrated in figures Fig. 3.9 for one qubit, and Fig. 3.4 and Fig. 3.5 for two qubits. To perform a measurement, see Sec. 2.3.3 for the usual tiles, and roll a d100 for the new tiles: if the result is between 1 and 50, transform states ◓, ◒ into ○; if the result is between 51 and 100, transform states ◓, ◒ into ●.

Finally, let us make an observation. If we have a state with property *white* or *black*, it will have property *left* with 50% probability, and property *right* with 50% probability. It will also have property *up* with 50% probability, and property *down* with 50% probability. That is, the three bases {○, ●}, {◐, ◑}, {◓, ◒} are *maximally incompatible*: a state in one basis looks completely random when measured in either of the two. The interesting point is that for single-qubit systems, or single squares in QTris, the maximum number of maximally incompatible bases is exactly three. Any attempt of producing a fourth basis inevitably produces an overlap with one of the three.

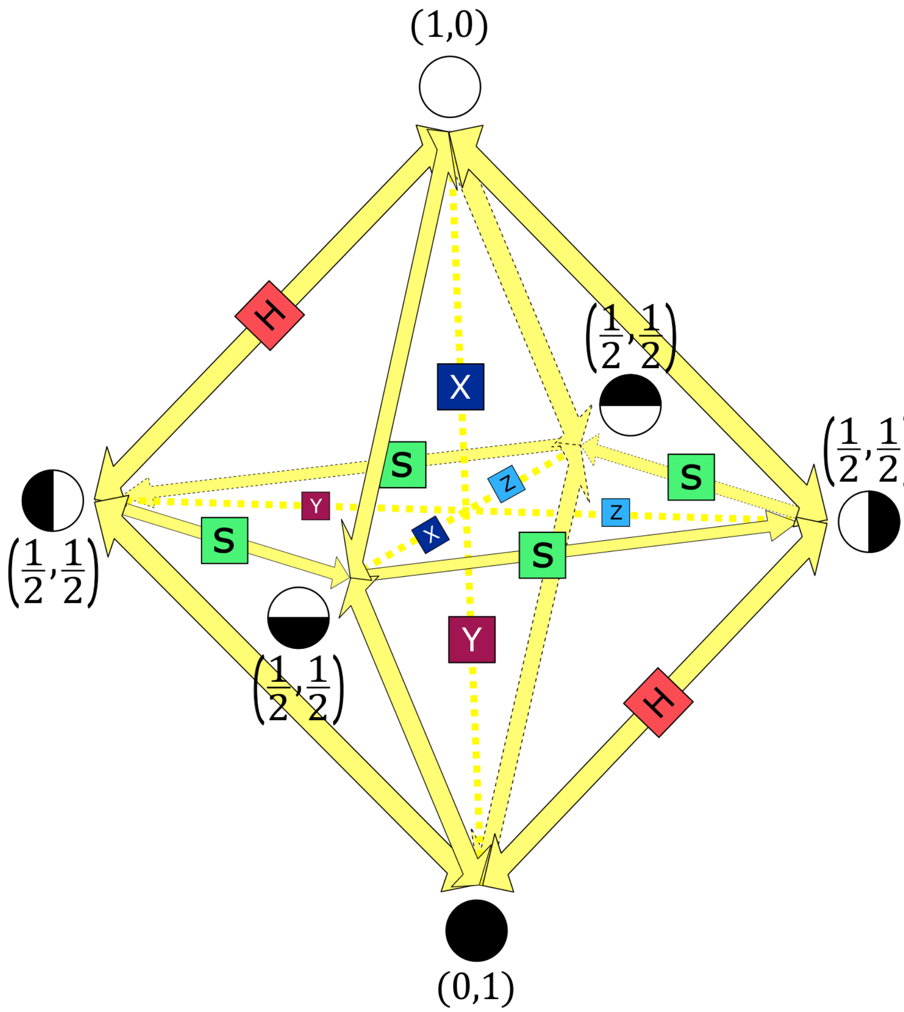


Figure 3.9: **Octahedral map of single-square operations** X, Y, Z, H, S **and of measurement probabilities.** The numbers in the parentheses (x, y) next to the symbols ○, ●, ◐, ◑, ⛀, ◒ represent the probabilities of obtaining ○ and ● respectively after performing a measurement.

3.7 Game variant: Eve*

In this variant of the game, there are three players: Alice, Bob and Eve. The game is cooperative, as Alice and Bob try to make as many qtris as possible together, while Eve tries to hinder their efforts. The game consists in several sets and it ends as soon as one of the two parties (Alice and Bob vs. Eve) makes three points.

Game Phases Summary [Eve Variant]

1. Preparation phase
   (a) Fill the entire game grid with valid states.
   (b) Distribute four (4) cards to each player (Alice, Bob, and Eve).
   (c) Determine the order of play (dice roll). The playing order must be Alice-Eve-Bob.
   (d) Mulligan: each player draws $n-1$ cards, where n is the number of discarded cards.
2. Operations phase
   - Players perform, respecting the previously established playing order, a total of five (5) alternating game turns.
     (a) Structure of each game turn for Alice and Bob:
       i. Draw one (1) card from the operations card deck.
       ii. Play one (1) operations card from the hand.
     (b) Structure of each game turn for Eve:
       i. Draw two (2) cards from the operations card deck.
       ii. Play two (2) operations cards from the hand, or, if not already done in the current game, discard two cards from the hand and perform the measurement of a state on the grid in the preferred game basis.
3. Measurement phase
   - Perform the measurement procedure of the game grid as reported in the box in Sec. 2.3.3 or Sec. 3.3.1.
4. Scoring phase
   - Alice and Bob receive as many points as the qtris they have made minus one. Example: Alice and Bob end the game with three (3) qtris in their favor. They will receive, in this case, two (2) points.
   - Eve receives one (1) point only if Alice and Bob have concluded the game with fewer than two (2) qtris.

3.8 And now?

Congratulations! You've reached the end of the QTris rulebook. Now you can play and have fun.

Those who have understood the rules, played the game, and read the quantum mechanics leaflets, will have gained quite a good understanding of how this physical theory works. Those who have only played will still have learned quite a bit about quantum mechanics —– even without realizing it! Many popular science books on quantum mechanics are sometimes very confusing, but the excellent book by Terry Rudolph [Rud17] is very insightful. QTris players will find it very interesting and attuned with the QTris game.

Finally, those who have read and studied the second part of this book with the additional chapters on quantum mechanics can truly say they know the basics of quantum mechanics — at a level that may even be useful for future university studies.

Hopefully some of you will feel the urge to deepen their knowledge. The following chapters provide a rigorous and precise introduction to quantum mechanics, where the reader will recognize that playing QTris is essentially performing quantum calculations. If you want to solve more QTris problems and see the equivalent calculations in quantum mechanics, look on the arXiv for the paper called Problems in QTris [DeS25].

QM

QTris and quantum mechanics. We have reached the end of this journey (for now). We have seen many examples and quantum experiments that can be played in QTris. Moreover, every QTris game corresponds to a quantum mechanics experiment that we can conduct in a laboratory!
At this point, a question arises. Does *everything* that can be done in quantum mechanics have a counterpart in QTris? The answer is, without a doubt, *yes!* However, the richness of quantum mechanics requires many more states and cards to efficiently represent other states, along with new probability tables.
The wonderful thing is that anyone can learn to create new tiles, cards, and probability tables to perform new quantum experiments. To learn how to do all this, an interested player has two options: take a course in quantum information mechanics or get their hands on the new game **Escape from the black hole** [AH25]!

The chapters with the mathematical formalism of quantum mechanics are removed in this online version and are available in the full book.

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